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**R,Q INVENTORY PROBLEM WITH  
UNKNOWN MEAN DEMAND  
AND LEARNING (A SEQUEL)**

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R,Q, INVENTORY PROBLEM WITH UNKNOWN MEAN DEMAND AND  
LEARNING (A SEQUEL)

TECHNICAL REPORT

BY

ALAN J. KAPLAN

SEPTEMBER 1977

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## Abstract (cont)

results of simulation using real world demand data, which proved favorable to this approach. It discusses a modification to current approaches for determining cost of backorder.

This report was motivated by concern with management of low failure items. For such items the discounted cash flow structure is most appropriate.

## INTRODUCTION

This report is a sequel to an earlier paper by the same author [5]. We address a single product, continuous review model with stationary Poisson demand, fixed lead time, and backorders. Such a model has been effectively studied when mean demand is known. However, we are concerned with managing new items for which only a Bayesian prior distribution on the mean is available. As demand occurs, the prior is updated and our control parameters are revised. These include the reorder point and reorder quantity.

In the earlier paper, a heuristic solution to finding an optimum reorder point, given the reorder quantity, was presented. It was justified analytically, and validated by simulations using randomly generated data.

This report introduces an alternative cost structure, discounted cash flow, and adapts the heuristic approach to this cost structure (Chapter I). It reports on the results of simulation using real world demand data, which proved favorable to this approach (Chapter II). It discusses a modification to current approaches for determining cost of backorder (Chapter III).

This report was motivated by concern with management of low failure items. For such items the discounted cash flow structure is most appropriate.

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## CHAPTER I

### DERIVATION OF COMPUTATIONAL PROCEDURE

#### 1.1 Cost Structure

Costs considered are purchase costs and backorder costs. Each time a buy of  $Q$  units is made, costs of  $C_p + (\mu)(Q)$  are incurred where  $\mu$  is unit price. Backorder costs are time weighted so that each additional day a unit is on backorder, a cost of  $C_b$  is incurred.

Costs are discounted to account for the time value of money and the probability of obsolescence. The discount rate for a cost incurred at time  $t$  is  $e^{-it}$ . In addition, looking at time  $t$  in the future, there is a probability of  $e^{-ot}$  the item will not be obsolete by then. Hence the discounted, expected value of a cost  $C(t)$  which would be incurred in the absence of obsolescence is  $C(t) e^{-it} e^{-ot} = C(t) e^{-(i+o)t}$ . Thus, impact of time can be accounted for by a discount factor  $\alpha$  equal to  $(i+o)$ .

Holding costs are not considered because the largest elements of military holding costs are financial and obsolescence cost, and these are more correctly considered as discussed than as a linear holding cost [6]. For low failure items under uncertainty, use of linear holding costs is particularly inappropriate. It is quite possible for the initial purchase to remain on hand for many years, in which case, using linear holding costs, a total holding cost significantly greater than the price of the item is computed, whereas the only cash outlay besides purchase is a small storage cost.

#### 1.2 Basic Approach

As in the earlier paper, we find the difference in cost,  $\Delta R$ , between ordering at  $R$ , or delaying the order until assets fall to  $(R-1)$ . Optimum  $R$  then satisfies.

$$(1.2.1) \quad \Delta R^* \leq 0$$

$$\Delta(R^*+1) > 0$$



We think of stock being issued in the order in which it is bought. Conceptually, costs incurred in backordering a demand are associated with the buy that provides the unit which eventually satisfies the demand, as described more fully in Silver [8] or Kaplan [7].

Letting

$\lambda$  - demand rate

$L$  - lead time

$B_{\alpha}(R, Q | \lambda, L)$  - discounted expected value of backorders associated with a buy of  $Q$  units at level  $R$ . Backorders are discounted back to the time of order.

$t$  - time until next demand

$f(t; \lambda)$  - distribution on  $t$

Then

$$(1.2.2) \quad \Delta R(Q, \lambda, L) = C_p + (u)(Q) + C_b B_{\alpha}(R, Q | \lambda, L) - \int_0^{\infty} e^{-\alpha t} f(t; \lambda) [C_p + (u)(Q) + C_b B_{\alpha}(R-1, Q | \lambda, L)] dt$$

For example, if ordering at  $R$  means ordering at time  $t_0$ , then ordering at  $R-1$  means ordering at time  $t_0 + t$ .  $B(R-1, Q | \lambda, L)$  is then the value of backorders associated with that order, discounted back to time  $t_0 + t$ . For comparability, it must be further discounted (multiplied by  $e^{-\alpha t}$ ) back to time  $t_0$ .

Note that the term in brackets under the integral sign in (1.2.2) is actually independent of the variable of integration. Hence

$$(1.2.3) \quad \Delta R(Q, \lambda, L) = C_p + (u)(Q) + C_b B_{\alpha}(R, Q | \lambda, L) - [C_p + (u)(Q) + C_b B_{\alpha}(R-1, Q | \lambda, L)] \left[ \int_0^{\infty} e^{-\alpha t} f(t; \lambda) dt \right]$$

To find  $R^*$ , we integrate (1.2.3) over the prior on  $\lambda$ . Computationally, this is approximated by using a histogram to approximate the prior:

$$\int \Delta R(Q, \lambda, L) d\lambda \approx \sum_1 \Delta R(Q, \lambda_1, L) g(\lambda_1)$$

### 1.3 Integral Solution

Since demand is Poisson distributed,  $f(t;\lambda)$  is exponential with mean  $1/\lambda$ , Hence

$$(1.3.1) \quad \int_0^{\infty} e^{-\alpha t} f(t;\lambda) dt = \int_0^{\infty} e^{-\alpha t} \lambda e^{-\lambda t} dt = \frac{\lambda}{\lambda + \alpha}$$

### 1.4 Solution for $B_{\alpha}(R, Q | \lambda, L)$

The following is taken from unpublished work by W. Karl Kruse.

Consider each of the  $Q$  units separately and index them by  $i$ ,  $i \rightarrow 1, 2, \dots, Q$ . Let  $t_i$  be the time until the  $i^{\text{th}}$  unit is demanded, measured from the time the order is placed. If  $t_i < L$ , then the  $i^{\text{th}}$  unit has a backorder associated with it. This backorder is cleared up at  $L$ . For  $t_i < L$ , the discounted time on backorder is:

$$(1.4.1) \quad \int_{t_i}^L e^{-\alpha t} dt = \frac{e^{-\alpha t_i} - e^{-\alpha L}}{\alpha}$$

Now  $t_i$  is equal to the time until the  $R+i^{\text{th}}$  demand since the  $R$  assets existing at the time of buy must first be consumed before any of the  $Q$  units are used. Since demand is Poisson,  $t_i$  has a Gamma distribution with parameters  $R+i$  and  $\lambda$  where Gamma is denoted:

$$(1.4.2) \quad g(x; a, b) = \frac{e^{-bx} x^{a-1} b^a}{\Gamma(a)}$$

Using (1.4.1)

$$(1.4.3) \quad B_{\alpha}(R, Q | \lambda, L) = \sum_{i=1}^Q \int_0^L g(t_i; R+i, \lambda) \frac{[e^{-\alpha t_i} - e^{-\alpha L}]}{\alpha} dt_i$$

Now, using (1.4.2)

$$(1.4.4) \quad e^{-\alpha t_1} g(t_1; R+1, \lambda) = \frac{e^{-(\lambda+\alpha)t_1} t_1^{R+1-1} \lambda^{R+1}}{(R+1)!}$$

$$= \frac{\lambda^{R+1}}{(\lambda+\alpha)^{R+1}} g(t_1; R+1, \lambda+\alpha)$$

Substituting (1.4.4) into (1.4.3), and denoting the cumulative distribution function of the Gamma by  $G(X, a, b)$ ,

$$(1.4.5) \quad B_{\alpha}(R, Q | \lambda, L) = \sum_{i=1}^Q \left[ \frac{\lambda^{R+1}}{(\lambda+\alpha)^{R+1}} G(L; R+1, \lambda+\alpha) - e^{-\alpha L} G(L; R+1, \lambda) \right]$$

For computation purposes, we may use the well known equivalence:

$$(1.4.6) \quad G(X; a, b) = P(a; bx)$$

where  $P$  is the Poisson complementary cumulative distribution function and  $a$  is integer.

### 1.5 Histogram Approximation

The prior, for convenience, is assumed to be Lognormal, although in the earlier paper we assumed Gamma. We are not being too inconsistent since for most parameter values the Gamma and Lognormal closely resemble each other.

For the Lognormal [4],

$$(1.5.1) \quad f(x; u, \sigma) = \frac{1}{\sigma x \sqrt{2\pi}} \exp \left[ -\frac{1}{2\sigma^2} (\log x - u)^2 \right]$$

$$\sigma^2 = \log \left[ \frac{\text{Var}(x)}{E_x^2(x)} + 1 \right]$$

$$u = \log E_x(x) - \sigma^2/2$$

The following additional results are derived in the Appendix.

$$(1.5.2) \quad \int_{a_i}^{a_{i+1}} f(x) dx = \int_{a'_i}^{a'_{i+1}} g(y) dy$$

$$(1.5.3) \quad \int_{a_i}^{a_{i+1}} x f(x) dx = \int_{a'^{-\sigma}_i}^{a'^{-\sigma}_{i+1}} g(y) dy \cdot E_x(x)$$

where  $a'_i = (\log a_i - u)/\sigma$  and  $g(y)$  denotes the Normal density for mean 0, variance 1.

Now, we would like to develop a 10 cell histogram to approximate the Lognormal. For given cell boundaries  $a_i$ ,  $i = 1 - 11$ , we have from (1.5.2)

$$\text{Probability (i}^{\text{th}} \text{ cell)} = \int_{a_i}^{a_{i+1}} f(x) dx = G(a'_{i+1}) - G(a'_i)$$

$$\text{Value (i}^{\text{th}} \text{ cell)} = \frac{\int_{a_i}^{a_{i+1}} x f(x) dx}{\int_{a_i}^{a_{i+1}} f(x) dx} = E_x(x) \frac{G(a'^{-\sigma}_{i+1}) - G(a'^{-\sigma}_i)}{G(a'_{i+1}) - G(a'_i)}$$

where  $G(x)$  is the cumulative unit normal distribution.

To choose values for the  $a_i$ , we used the relationship  $a_i = \exp(\sigma a'_i + u)$ , i.e. the inverse of the relationship for going from  $a_i$  to  $a'_i$ . We always chose for the  $a'_i$ ,  $i = 1 - 11$ , these values:

|        |        |        |        |        |        |                                  |      |
|--------|--------|--------|--------|--------|--------|----------------------------------|------|
| -5.0   | -3.2   | -2.4   | -1.6   | -.8    | 0.     | +.8                              | +5.0 |
| $a'_1$ | $a'_2$ | $a'_3$ | $a'_4$ | $a'_5$ | $a'_6$ | $a'_7 \dots \dots \dots a'_{11}$ |      |

Note that the  $a'_i$  are actual cell boundaries for integrating the unit normal. The values shown were found after some experimentation.

After experimenting with this approach it was found that the histogram means were close to the true mean,  $E_x(x)$ . The histogram standard deviations (based on values for  $i^{\text{th}}$  cell, probability of  $i^{\text{th}}$  cell,  $i = 1 - 11$ ) were only a fraction of the true standard deviation. This fraction depended on the true coefficient of variation,  $\gamma$ , but given  $\gamma$ , were independent of the true mean or variance. The fractional values are:

|          |      |      |      |      |      |      |      |      |      |      |      |      |      |
|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $\gamma$ | .01  | 0.1  | 0.2  | 0.4  | 0.6  | 0.8  | 1.0  | 1.2  | 1.4  | 1.6  | 1.8  | 2.0  | 2.5  |
| Fraction | .977 | .974 | .974 | .972 | .970 | .967 | .963 | .958 | .953 | .947 | .940 | .932 | .911 |

Thus, if the true  $\gamma$  were 1.4, we would find that histogram standard deviation  $\div$  true standard deviation was .953. To partially correct for this, if  $\gamma$  were 1.4 we would first divide the true standard deviation by .953 and then calculate the histogram for this revised value.

## CHAPTER II

### SIMULATION

#### 2.1 Purpose of Simulation

The purpose of the simulation was to determine whether the procedure derived would work effectively in a real world environment, in the management of a particular class of items. The items of concern are termed "insurance." They are highly essential with very low failure rates. When first introduced into the system, there is no failure rate forecast available for insurance items. Instead, a prior distribution is developed based on the mean and variance of the failure rates over the existing catalog of insurance items.

The inventory model originally proposed to manage insurance items [1] was used as the standard of comparison by which to judge the new procedure. This inventory model, in form, is that appropriate for Poisson demand with known mean under steady state conditions and linear holding and backorder costs, not discounted. Uncertainty about the true mean is accounted for by using a Compound Poisson distribution for demand. In particular, the prior is assumed to be Gamma, which means that the posterior distribution of demand about the forecasted mean is Negative Binomial. This model represented the state of the art when our earlier paper was written.

#### 2.2 Data Base

The data base used was that used to validate the current policy [2]. It consists of four years of history for 630 insurance items, for the years 1967 to 1971, obtained from what is now called US Army Missile Readiness Command.

There is one important respect in which the data is treated differently in this study than it was in the original study. All requisitions are assumed to be for one unit regardless of what the historical quantity requested was. This "rewriting" of history is based on the fact that insurance items by definition should not normally have requisitions for more than one each. Currently, there is a recommendation being considered to establish a Maximum Release Quantity of one for insurance items except in special cases.

Following are the means and variances of the catalog based on the first two years of the data, computed with and without the historical requisition quantity:

|          | <u>Historical<br/>Qtys</u> | <u>Qtys Set To<br/>1</u> |
|----------|----------------------------|--------------------------|
| Mean     | 1.85                       | .79                      |
| Variance | 28.3                       | 2.1                      |

The data base has two additional drawbacks besides the problem already described. Because of the way it was developed, all items in the data base were classified as insurance at the end of the third year of data; i.e., any item which had become obsolete between the first and third years would not show up in our catalog at all. Secondly, for reasons not known, the 4th year of data reflected a significantly lower overall demand rate as shown below:

| Year     | 1    | 2    | 3    | 4   | Avg:<br>of 3<br>Years | Overall<br>Average |
|----------|------|------|------|-----|-----------------------|--------------------|
| Mean     | .80  | .77  | .83  | .39 | .80                   | .70                |
| Variance | 1.84 | 2.29 | 1.81 | .98 | 1.98                  | 1.73               |

### 2.3 Simulation Methodology

All items were started with on hand equal to their R+Q values. The Q values used were the classical EOQ, except that during the first year only 30% of EOQ was used. All of this basically corresponds to what is currently being done.

Actual demands were taken from the historical record, parameters updated after each demand, and buys placed as necessary. Each item's procurement lead time was recorded in the data base and these were assumed to be accurate.

Two sets of mean/variance pairs were used for the prior. The 3 year catalog averages were used until an item was 2.5 years old. Then the 4th year catalog averages were used to reflect the drop off in demand to be

expected in the 4th year. An alternative would have been to discard the 4th year entirely, but we felt it would be more desirable to use all the data available.

It was assumed all items had a unit price of \$1000. The advantage of this is that no one or two expensive items can then dominate the simulation results. Previous work [1] had found no correlation between unit price and item behavior. If such a relationship did exist, but were not detected, it would undoubtedly be that higher unit price items had lower demands than were reflected in the catalog. If this were so, the baseline policy, which tends to overstock, would be hurt most.

The cost structure assumed was that commonly used to evaluate alternative inventory policies within US Army Materiel Development & Readiness Command [3]. This structure is essentially consistent with the baseline policy but not with the proposed policy and so should favor the baseline.

Costs include backorder costs, holding costs and "excess" costs. Backorder costs are per requisition on backorder per year. Holding costs are per dollar of inventory on hand per year. Excess costs represent the difference between the purchase price of assets on hand or on order at the end of the simulation and the calculated worth of these assets. Since excess is calculated this way, it is excluded from holding cost charges. Thus, MIRCOM's holding cost rate is 23% including 10% for obsolescence; in charging holding cost in the simulation, a rate of 13% is used.

An algorithm is used to calculate worth of end of simulation assets for each item [3]:

a. Item's end of simulation demand rate,  $D$ , is estimated. If  $D = 0$ , worth is set to 0.

b. End of simulation requirements objective ( $R+Q$ ) is computed. If assets  $\leq$  requirements objective, worth is set equal to purchase price (no obsolescence).

c. If assets exceed requirements objective, worth of all assets ( $A$ ) above requirements objective is calculated as:

$$(UP)(D) \sum_{j=1}^N \left( \frac{1-\theta}{1+i} \right)^{j-.5} + (UP)(A-ND) \left( \frac{1-\theta}{1+i} \right)^{N+.5}$$

where  $i$  is interest rate,  $\theta$  is obsolescence rate and  $N$  is the largest integer



such that  $A \geq ND$ . Essentially, each item is given a worth of  $(UP) \left( \frac{1-\theta}{1+i} \right)^{t-.5}$  where  $t$  is the year in which it would be issued assuming constant demand rate of  $D$ , no obsolescence. Note that  $\left( \frac{1-\theta}{1+i} \right)^t$  is discrete form of  $e^{-(1+\theta)t}$ . Included in  $\theta$  were the small charges for storage (1%) and losses (2%) + 10% obsolescence. Since  $i = 10\%$ , discount factor was .87/1.1.

One problem in applying this methodology is how to estimate  $D$  and compute requirements objectives. Another problem is that as the data base was formulated, there really was little chance for items to become obsolete and no way to distinguish these items.

These problems were solved in two different ways, so we had two different comparisons of results. In Method I,  $D$  was based only on the last 2 years of history, so if 0, we were effectively treating item as obsolete (this is method for getting  $D$  used in the past in other contexts [3]). In Method II,  $D$  was based on all four years of history plus the prior. No items could be considered obsolete although an excess cost was still computed when assets exceeded requirements objective, with obsolescence rate set to 0; i.e. factor used was .97/1.1. Since no obsolescence rate was charged under Method II, holding cost rate input to the two inventory models, baseline and proposed, was only 13%. Under both Method I and Method II end of simulation requirements objective was based on the baseline model assuming Poisson demand regardless of which model was being simulated, so that a given asset would be costed out the same way regardless of inventory policy.

Since backorders were not discounted for time of occurrence in the cost evaluation, we did not discount for interest in applying the proposed procedure. We did discount for obsolescence when running under Method I.

Cost of backorders was assumed to be \$23,000 under Method I and \$13,000 under Method II. Under the baseline policies target stock availability is set to

$$100\% - \frac{(\text{Hold Cost\%})(\text{Unit Price})}{(\text{Backorder Cost Rate})}$$

as part of the optimizing procedure [3]. Thus, the backorder cost rates assumed implied 99% availability target under the baseline policy and that

is why they were chosen. For how backorder costs would be computed in the real world, see Chapter III.

#### 2.4 Results

Shown below are the cost comparisons under both Methods I and II for baseline policy [1] vs proposed policy (Chapter I).

| POLICY    | HOLDING COST | EXCESS | BO COST | TOTAL |
|-----------|--------------|--------|---------|-------|
| Method I  |              |        |         |       |
| Baseline  | 1628.        | 1758.  | 407.    | 3793. |
| Proposed  | 1055.        | 1161.  | 1488.   | 3704. |
| Method II |              |        |         |       |
| Baseline  | 1634.        | 777.   | 232.    | 2644. |
| Proposed  | 1064.        | 305.   | 826.    | 2194. |

Under Method I, proposed policy showed a 6% improvement, and under Method II, improvement was 17%. Because of the data base problems, it seems reasonable to conclude only that the proposed policy does in fact work, without trying to conclude just how much better it is. When runs were made using only one set of catalog parameters, based on all 4 catalog years, results were comparable under Method II, but under Method I improvement narrowed to 2.4%.

Just to confirm that the difference between Method II and Method I was not due to differences in the backorder cost parameter, Method II was rerun with \$23,000 cost of backorder. Results were like those shown.

#### 2.5 Use of Simple Poisson

The baseline model without any compensation for uncertainty about the mean was tested; i.e., the simple Poisson distribution was assumed. This is easier to implement than the learning model, and our earlier work with the learning model had indicated that attempting to compensate for uncertainty simply by use of the compound Poisson could do more harm than good.

Results follow:

|           | Policy   | Holding Cost | Excess | B.O. Cost | Total |
|-----------|----------|--------------|--------|-----------|-------|
| Method I  |          |              |        |           |       |
|           | Baseline | 1628.        | 1758.  | 407.      | 3793. |
|           | Poisson  | 1238.        | 1279.  | 1120.     | 3637. |
|           | Proposed | 1055.        | 1161.  | 1488.     | 3704. |
| Method II |          |              |        |           |       |
|           | Baseline | 1634.        | 777.   | 232.      | 2644. |
|           | Poisson  | 1244.        | 442.   | 622.      | 2308. |
|           | Proposed | 1064.        | 305.   | 826.      | 2194. |

The Poisson fell between Baseline and Proposed as regards amount of inventory investment (note holding cost results). This is partly because the learning model may react to uncertainty by lowering stockage below what it would be in the certain case, and partly because the discounted cash flow model will give lower reorder point solutions than the steady state model, under certainty.

Poisson actually did better than learning model in Method I, worse under Method II. While it is our judgement, taking into account the results and the difficulties with the data base, that the learning model is superior, results are ambiguous enough to consider the simple Poisson for implementation.

### CHAPTER III

#### BACKORDER COSTS

There are two methods for assigning backorder costs used within Dept of the Army, the "Lambda" approach, and the "ERPSL" approach.

The "Lambda" approach [9] is used at wholesale level. It is assumed that the backorder cost per requisition backordered is independent of the item<sup>\*</sup> or quantity demanded. This cost, or "Lambda", is determined by its impact on projected budgetary needs and supply performance. Higher "Lambda's" results in both higher dollar requirements and higher supply performance. A "Lambda" is chosen each year which will compromise between spending goals and performance goals.

The "ERPSL" approach [10] is being implemented at retail for certain low demand, highly essential items and is being considered for application at wholesale level to insurance items. Under this approach the cost of backordering a part is equated to the cost of holding the next higher assembly which would be rendered useless if the part were needed and not available. This type of approach has been discussed in many places in the past and has been used by the Navy in support of their Polaris submarine program.

In the remainder of this chapter we provide a rationale for the ERPSL approach within the discounted cost structure. We then justify a modification. With no loss of generality we assume the next higher assembly is the end item as it often is for insurance items.

Let

u - marginal value per unit time of having the end item operable, assumed to be constant over life of item.

C - cost of end item

---

\* The Lambda structure permits differentiation by items but this is not done currently except that there is a different Lambda for stock fund items than for PA items.

Then, assuming there is no production constraint,

$$(3.1) \quad C = \int_0^{\infty} u e^{-\alpha t} dt$$

where  $\alpha = (\theta+1)$ . By algebra,

$$(3.2) \quad u = C / \int_0^{\infty} e^{-\alpha t} dt = \alpha C$$

Thus, the value lost by deadlining the end item for lack of a part is at a rate of  $\alpha C$  per unit time, or unit price of end item holding cost rate.

There is a need to modify this result. While the purchase price is incurred at time  $t = 0$ , the use of the item does not begin until time  $t = 0 + \text{PLT}$ , where PLT is procurement lead time. One can argue about the extent to which the purchase cost is really incurred when the end item is ordered versus when it is received. This does not really matter much, however, since in our reorder point models we also assume the full purchase price is incurred when the item is ordered, and the important thing is that we are consistent in our assumption.

We have now, corresponding to (3.1)

$$(3.3) \quad C = \int_{\text{PLT}}^{\infty} u e^{-\alpha t} dt$$

or, by algebra,

$$(3.4) \quad u = C \alpha e^{-\alpha \cdot \text{PLT}}$$

Using (3.4) to cost out a backorder instead of (3.2) has the effect of multiplying the cost by  $e^{\alpha \text{PLT}}$ . For  $\alpha = 23\%$ ,  $\text{PLT} = 1$  year, this factor is 1.26.

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## APPENDIX

### TWO LOGNORMAL IDENTITIES

Notation used is that of main text, Section 1.5. Also, let  $y = \log x$  where  $\log$  refers to natural logarithm.

$$\begin{aligned}
 (A1) \quad \int_{a_1}^{a_{i+1}} f(x) dx &= \int_{a_1}^{a_{i+1}} \frac{1}{x\sigma\sqrt{2\pi}} \exp \left[ -\frac{1}{2\sigma^2} (\log x - u)^2 \right] dx \\
 &= \int_{\log a_1}^{\log a_{i+1}} \frac{1}{e^y \sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2\sigma^2} (y-u)^2 \right] e^y dy \\
 &= \int_{\log a_1}^{\log a_{i+1}} g_1(y) dy
 \end{aligned}$$

where  $g_1(y)$  is density for Normal random variable with mean  $u$ , variance  $\sigma^2$ .

$$(A2) \quad \int_{a_1}^{a_{i+1}} x f(x) dx = \int_{\log a_1}^{\log a_{i+1}} \frac{1}{\sigma\sqrt{2\pi}} \exp \left[ -\frac{1}{2\sigma^2} (y-u)^2 \right] e^y dy$$

Looking at all exponential terms:

$$\begin{aligned}
 & -\frac{1}{2\sigma^2} [y^2 - 2uy + u^2 - 2y\sigma^2] \\
 &= -\frac{1}{2\sigma^2} [(y - (u+\sigma^2))^2 - (u+\sigma^2)^2 + u^2] \\
 &= -\frac{1}{2\sigma^2} [y - (u+\sigma^2)]^2 + u + \frac{\sigma^2}{2}
 \end{aligned}$$

Substituting in (A2)

$$\begin{aligned}
 (A3) \quad \int_{a_1}^{a_{1+1}} x f(x) dx &= \int_{\log a_1}^{\log a_{1+1}} \frac{1}{\sigma\sqrt{2\pi}} \exp \left[ u + \frac{\sigma^2}{2} \exp \left[ -\frac{1}{2\sigma^2} [y - (u + \sigma^2)]^2 \right] \right] dy \\
 &= \exp \left( u + \frac{\sigma^2}{2} \right) \int_{\log a_1}^{\log a_{1+1}} g_2(y) dy \\
 &= E_x(x) \int_{\log a_1}^{\log a_{1+1}} g_2(y) dy
 \end{aligned}$$

where  $g_2(y) \sim N(u + \sigma^2, \sigma^2)$

Now letting  $a'_1 = (\log a_1 - u)/\sigma$ , we can restate (A1) and (A3) as

$$(A4) \quad \int_{a_1}^{a_{1+1}} f(x) dx = \int_{a'_1}^{a'_{1+1}} g(y) dy$$

$$(A5) \quad \int_{a_1}^{a_{1+1}} x f(x) dx = E_x(x) \int_{a'_1 - \sigma}^{a'_{1+1} - \sigma} g(y) dy$$

where  $g(y)$  is distributed  $N(0, 1)$



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